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where X is some point between a and x. Hence, obviously,

$$\underset{x \doteq a}{\text{Lim.}} \frac{f_1(x)}{f_2(x)} = \underset{x \doteq a}{\text{Lim.}} \frac{f_1'(X)}{f_2'(X)}.$$

If we now assume $f_2''(a) \neq 0$, we have, by the theorem first proved,

$$\underset{x \doteq a}{\text{Lim.}} \frac{f_1(x)}{f_2(x)} = \underset{x \doteq a}{\text{Lim.}} \frac{f_1'(x)}{f_2'(x)} = \frac{f_1''(a)}{f_2''(a)}.$$

Obviously, by the same process, in case $f_2''(a) = 0$, but $f_2'''(a) \neq 0$,

$$\underset{x \doteq a}{\text{Lim.}} \frac{f_1(x)}{f_2(x)} = \frac{f_1'''(a)}{f_2'''(a)},$$

and so on for the higher cases.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

348. Proposed by PROFESSOR L. E. DICKSON, Ph. D., The University of Chicago.

Prove the following relation between Jacobi's symbols:

$$\left(\frac{d}{n}\right) = (-1)^{m(d-1)/2} \left(\frac{d}{2md-n}\right)$$

where d and n are positive odd numbers and $2md > n$.

I. Solution by the PROPOSER.

Replace each symbol by the value given by the generalized reciprocity theorem; then on the right apply

$$\left(\frac{2md-n}{d}\right) = \left(\frac{-n}{d}\right) = (-1)^{(d-1)/2} \left(\frac{n}{d}\right).$$

Hence the relation will be true if

$$\frac{n-1}{2}\cdot\frac{d-1}{2}\equiv\frac{m(d-1)}{2}+\frac{d-1}{2}\cdot\frac{2md-n-1}{2}+\frac{d-1}{2}\quad(\mathrm{mod}\ 2),$$

or,

$$\frac{(d-1)}{2}[m+md-n+1]\equiv 0\quad(\mathrm{mod}\ 2).$$

But the first factor is an integer, and the second is an even integer.

II. **Solution by BENJAMIN FRANKLIN YANNEY, A. M., Mount Union College, Alliance, Ohio.**

An implied assumption is that d and n are relatively prime to each other. It follows, then, that d and $2md-n$ are also relatively prime to each other.

1. By the reciprocity theorem, $\left(\frac{d}{n}\right)\left(\frac{n}{d}\right)=(-1)^{\frac{1}{2}(d-1).\frac{1}{2}(n-1)}$.

2. Multiply by $\left(\frac{-1}{d}\right)=(-1)^{\frac{1}{2}(d-1)}$, member by member:

$$\left(\frac{d}{n}\right)\left(\frac{n}{d}\right)\left(\frac{-1}{d}\right)=(-1)^{\frac{1}{2}(d-1).\frac{1}{2}(n-1)+\frac{1}{2}(d-1)}=(-1)^{\frac{1}{2}(d-1).\frac{1}{2}(n+1)}.$$

But, $\left(\frac{n}{d}\right)\left(\frac{-1}{d}\right)=\left(\frac{-n}{d}\right)=\left(\frac{2md-n}{d}\right)$, as is well known. [See (35) and (36), p. 60, Bachmann's *Neuere Zahlentheorie.*]

3. Therefore, $\left(\frac{d}{n}\right)\left(\frac{2md-n}{d}\right)=(-1)^{\frac{1}{2}(d-1).\frac{1}{2}(n+1)}$.

4. Multiply by $\left(\frac{d}{2md-n}\right)$:

$$\left(\frac{d}{n}\right)\left(\frac{2md-n}{d}\right)\left(\frac{d}{2md-n}\right)=(-1)^{\frac{1}{2}(d-1).\frac{1}{2}(n+1)}\left(\frac{d}{2md-n}\right);$$

$$\text{or, }\left(\frac{d}{n}\right)(-1)^{\frac{1}{2}(2md-n-1).\frac{1}{2}(d-1)}=(-1)^{\frac{1}{2}(d-1).\frac{1}{2}(n+1)}\left(\frac{d}{2md-n}\right).$$

5. Multiply by $(-1)^{\frac{1}{2}(2md-n-1).\frac{1}{2}(d-1)}$ and reduce:

$$\left(\frac{d}{n}\right)=(-1)^{md(d-1)/2}\left(\frac{d}{2md-n}\right).$$

6. $\therefore\left(\frac{d}{n}\right)=(-1)^{m(d-1)/2}\left(\frac{d}{2md-n}\right)$, since $(-1)^{md(d-1)/2}=(-1)^{m(d-1)/2}$, d being an odd integer.